

Group Analyses

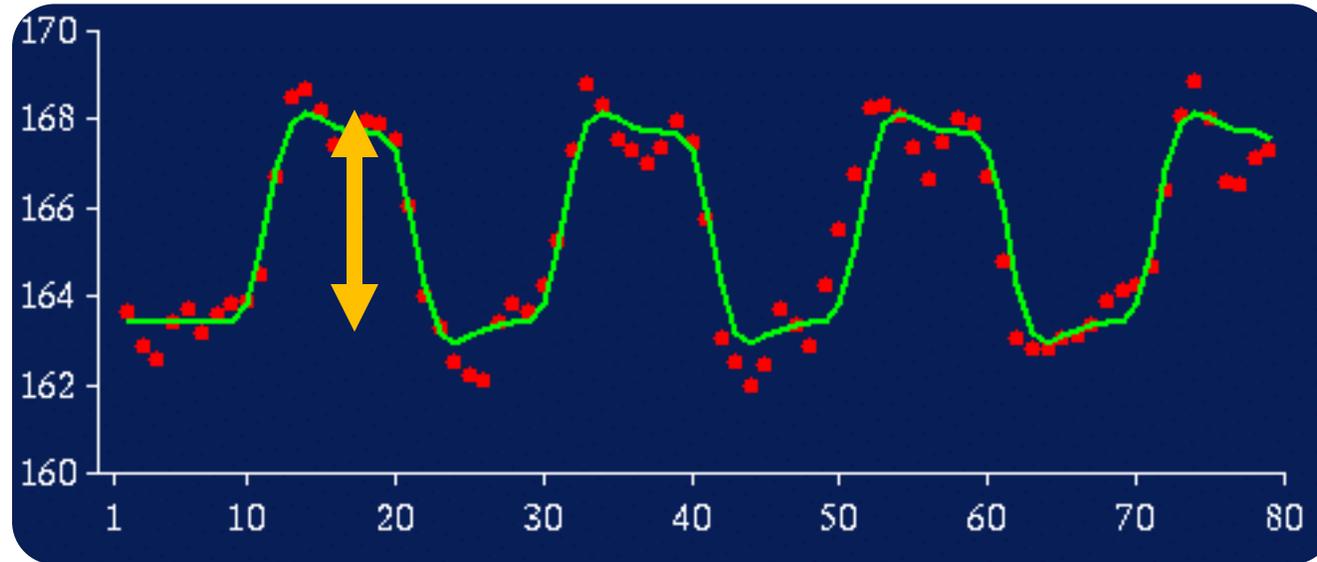
Emma Holmes



* Slides adapted from Will Penny

First Level: Subject 1

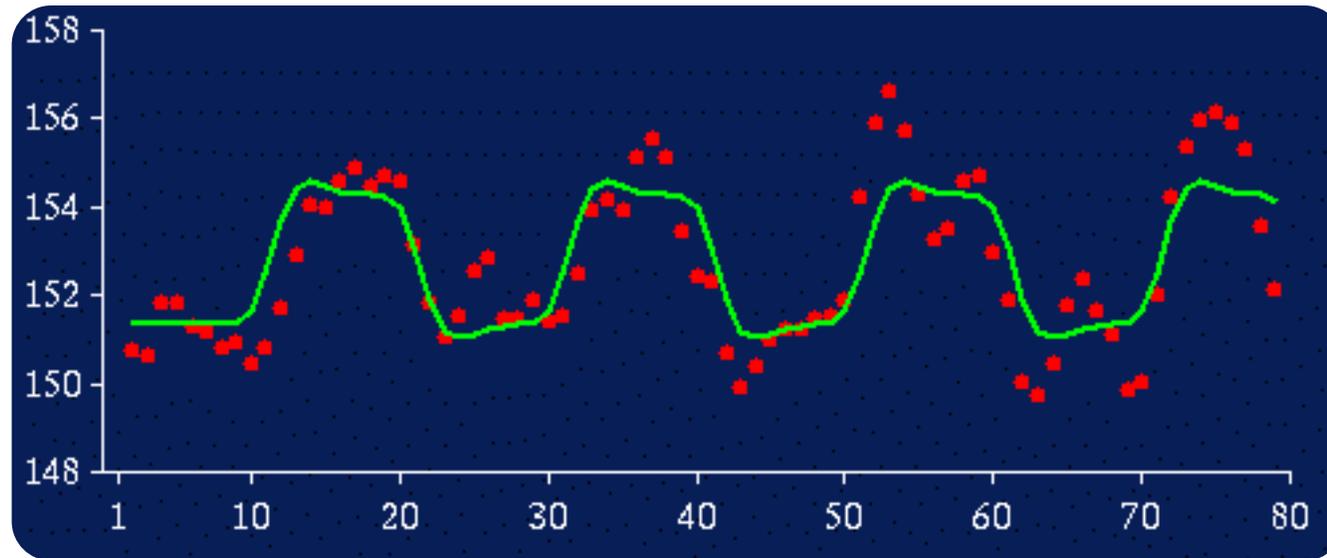
For voxel v in the brain



Effect size (c) ≈ 4

First Level: Subject 3

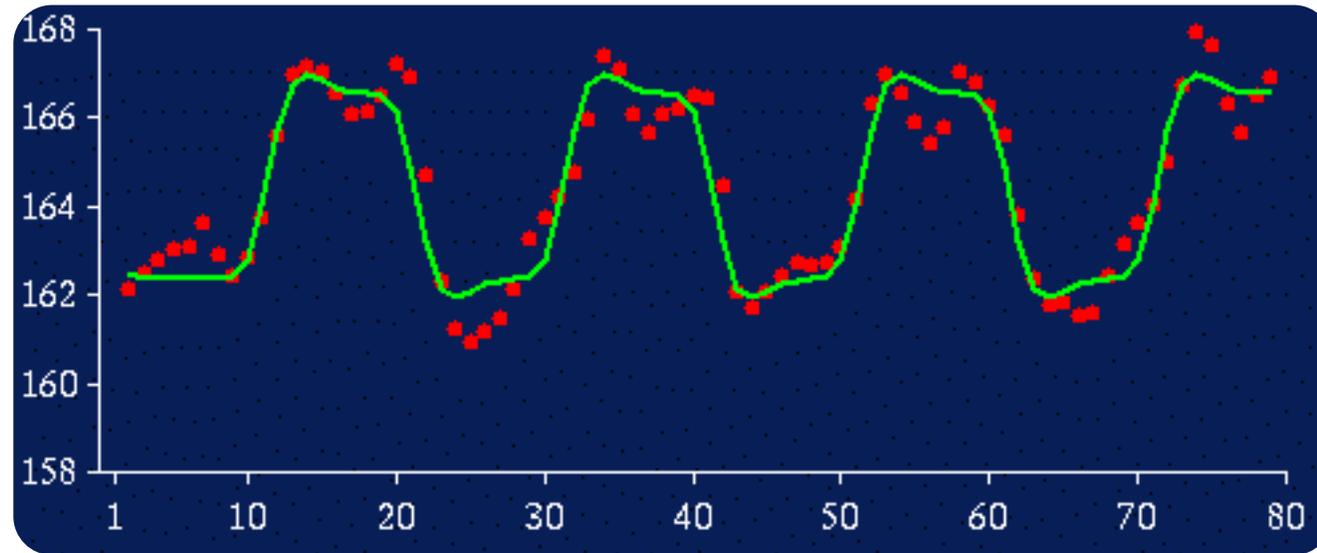
For voxel v in the brain



Effect size (c) ≈ 2

First Level: Subject 12

For voxel v in the brain



Effect size (c) ≈ 4

Second Level: Group Analysis

	c
Subject 1	4
Subject 2	3
Subject 3	2
Subject 4	1
Subject 5	1
Subject 6	2
Subject 7	3
Subject 8	3
Subject 9	3
Subject 10	2
Subject 11	4
Subject 12	4

Group effect (mean [m]) = 2.67

Between subject variability (stand dev [sb]) = 1.07

Standard error of the mean (SEM) = sb / \sqrt{N}
= 0.31

Is the effect significant at voxel v ? (one-sample t-test)

$$t = m / SEM = 2.67 / 0.31 = 8.61$$

$$p = 10^{-6}$$

This is called a Random Effects Analysis, because we compare the group effect to the between-subjects variability

Second Level: Group Analysis

	c
Subject 1	4
Subject 2	3
Subject 3	2
Subject 4	1
Subject 5	1
Subject 6	2
Subject 7	3
Subject 8	3
Subject 9	3
Subject 10	2
Subject 11	4
Subject 12	4

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Between subject variability (stand dev [sb]) = 1.07

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= 0.31

Is the effect significant at voxel v ? (one-sample t-test)

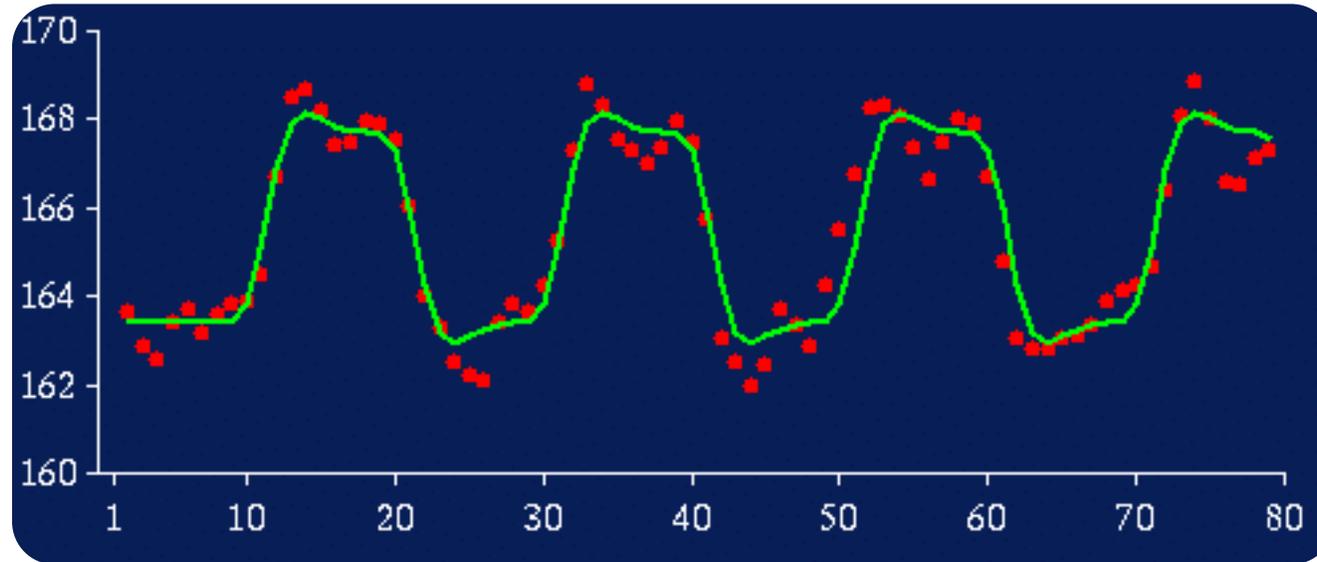
$$t = m / \text{SEM} = 2.67 / 0.31 = 8.61$$

$$p = 10^{-6}$$

...also known as the **SUMMARY STATISTIC** approach: We summarise the response of each subject by a single statistic (their effect size)

First Level: Subject 1

For voxel v in the brain



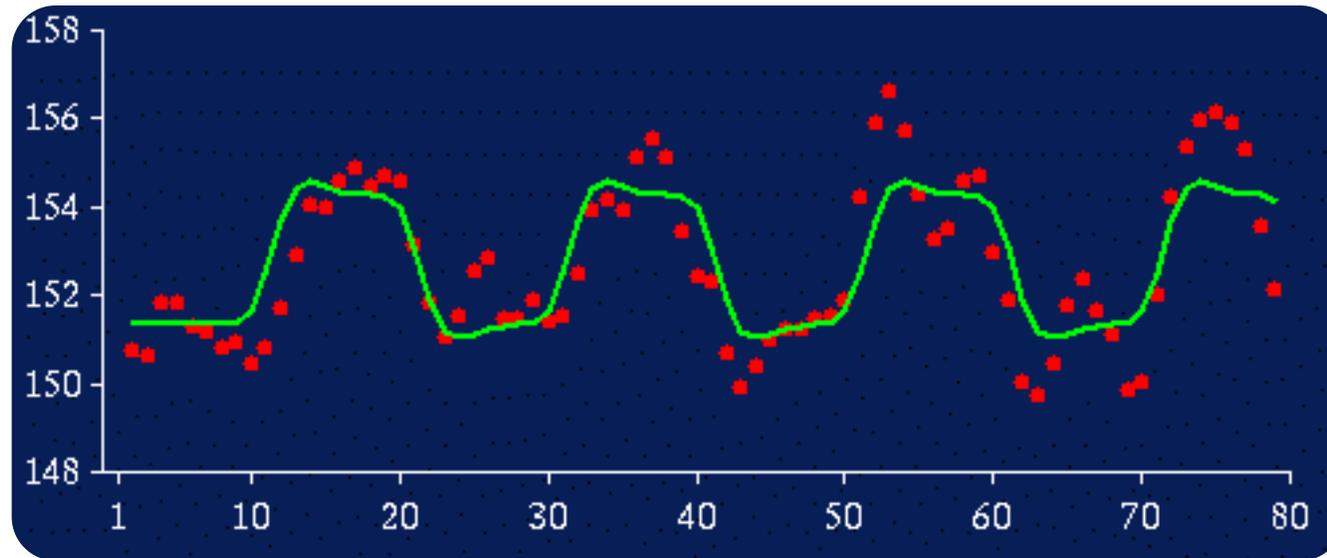
FIXED EFFECTS
ANALYSIS:
Not recommended for
neuroimaging data

Effect size (c) ≈ 4

Within subject variability (s_w) ≈ 0.9

First Level: Subject 3

For voxel v in the brain



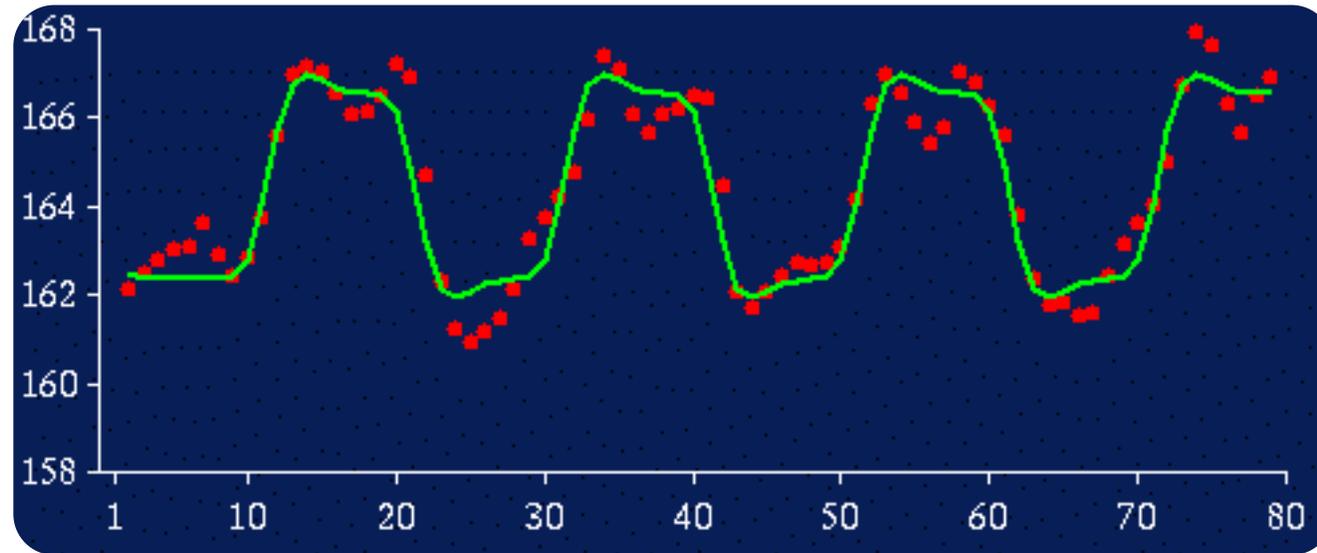
FIXED EFFECTS
ANALYSIS:
Not recommended for
neuroimaging data

Effect size (c) ≈ 2

Within subject variability (s_w) ≈ 1.5

First Level: Subject 12

For voxel v in the brain



FIXED EFFECTS
ANALYSIS:
Not recommended for
neuroimaging data

Effect size (c) ≈ 4

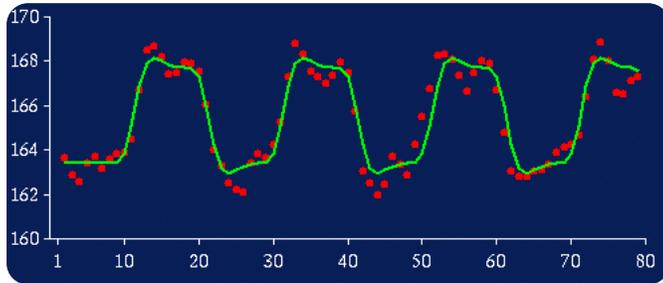
Within subject variability (s_w) ≈ 1.1

Fixed Effects Analysis

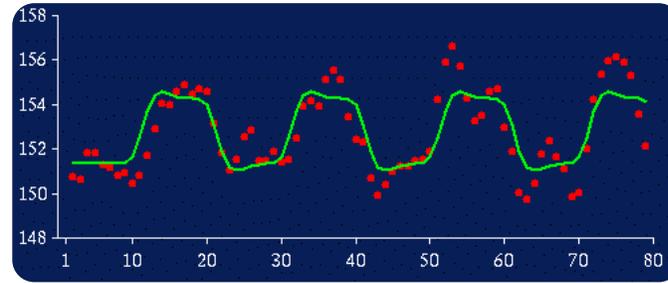
*FIXED EFFECTS
ANALYSIS:
Not recommended for
neuroimaging data*

Concatenate timeseries

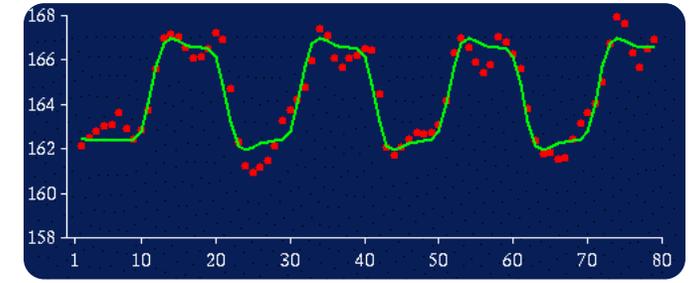
Subject 1



Subject 3



Subject 12



Each measurement is one scan from one subject
... we now have 600 scans (50 scans in each of 12 subjects)

We use this to calculate the average effect

Group Analysis: Fixed Effects

FIXED EFFECTS
ANALYSIS:
Not recommended for
neuroimaging data

	s_w
Subject 1	0.9
Subject 2	1.2
Subject 3	1.5
Subject 4	0.5
Subject 5	0.4
Subject 6	0.7
Subject 7	0.8
Subject 8	2.1
Subject 9	1.8
Subject 10	0.8
Subject 11	0.7
Subject 12	1.1

Group effect (mean [m])

= 2.67

Average within subject variability (sw)

= **1.07**

Standard error of the mean (SEMw)

= **sw /sqrt(N)**

= **0.04**

Is the effect significant at voxel v ?

$t = m/SEMw = 62.7$

$p = 10^{-51}$

Number of data
points is now total
number of scans
(i.e. 600)



Random Effects vs. Fixed Effects

Fixed Effects Analysis (FFX)

- We compare the group effect to the within-subject variability.
- It is an inference about this specific sample of subjects.
- Statistics are often inflated relative to random effects analysis.

Random Effects Analysis (RFX)

- We compare the group effect to the between-subject variability.
- It is an inference about the population from which the subjects were drawn: If you had a new subject from that population, you could be confident they would also show the effect.

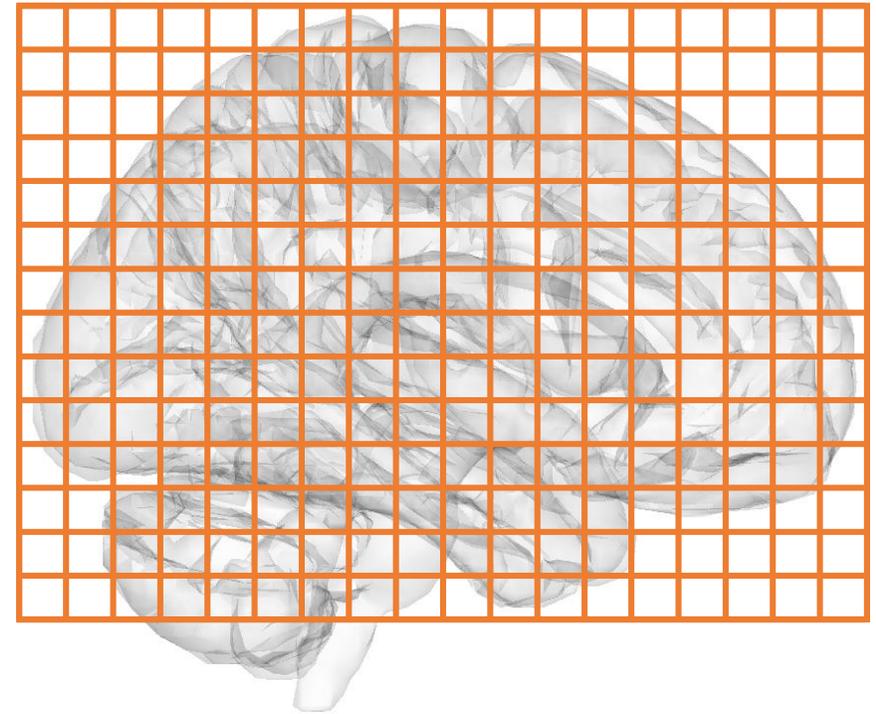
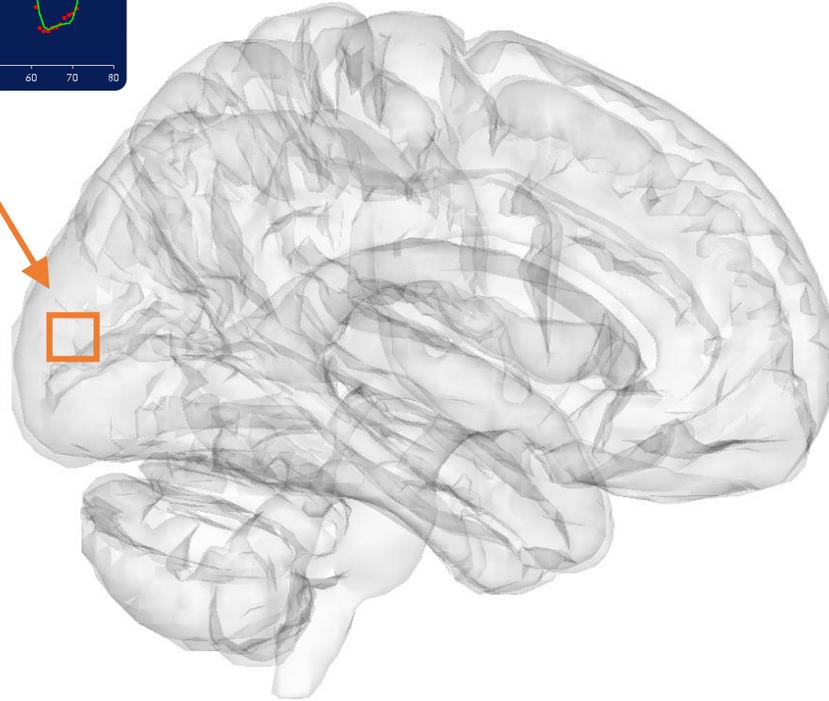
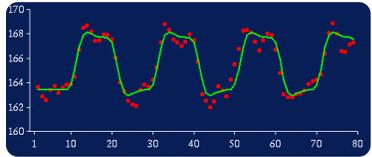
Random Effects vs. Fixed Effects

Mixed Effects Analysis (MFX)

- Has some random and some fixed effects.
- `spm_mfx`

Beyond a single voxel...

Voxel v



Random Effects: Summary Statistic

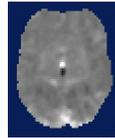
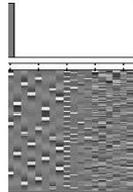
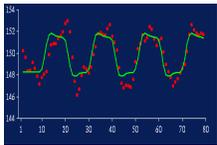
First level

Data (per voxel)

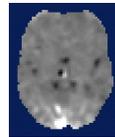
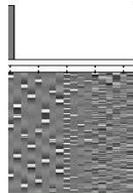
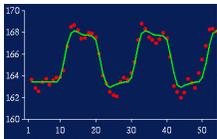
Design Matrix

Contrast Image

S1

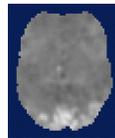
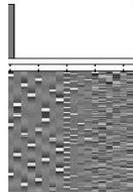
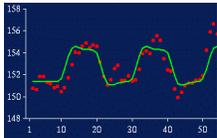


S2

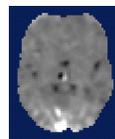
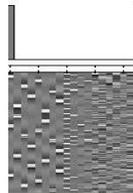
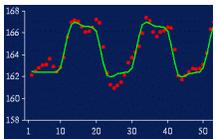


⋮

S11



S12



Random Effects: Summary Statistic

First level

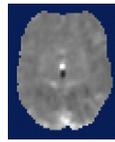
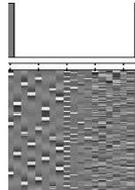
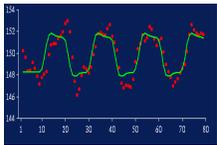
Second level

Data (per voxel)

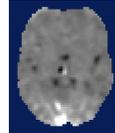
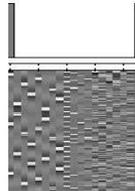
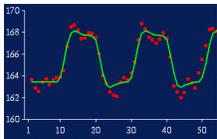
Design Matrix

Contrast Image

S1

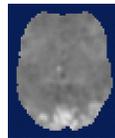
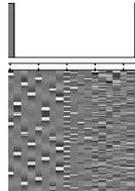
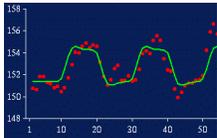


S2

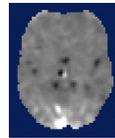
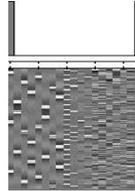
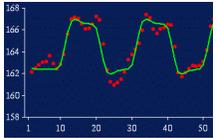


⋮

S11

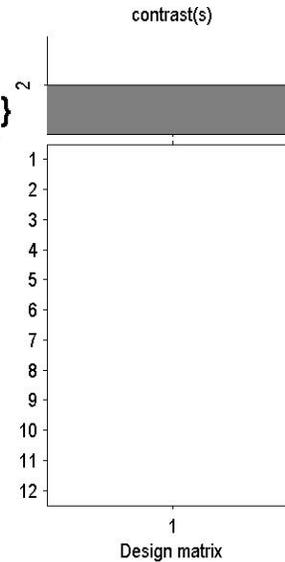


S12



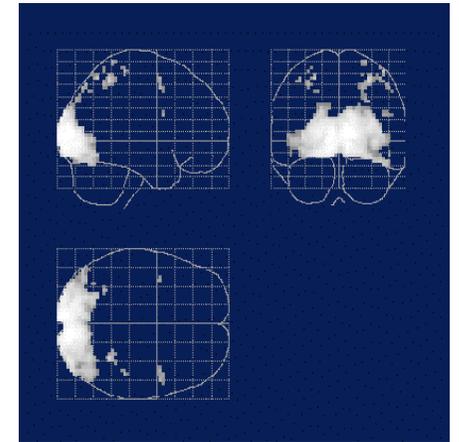
One-sample t-test

SPM{ T_{11} }



$$t = \frac{c^T \hat{\alpha}}{\sqrt{\text{Var}(c^T \hat{\alpha})}}$$

SPM(t)



Random effects:
summary statistic
approach

Hierarchical model

Level 1:

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$

Level 2:

$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$

\vdots

Level n :

$$\theta^{(n-1)} = X^{(n)}\theta^{(n)} + \varepsilon^{(n)}$$

At each level, the distribution of parameters is dependent on the level above

Multiple variance components at each level

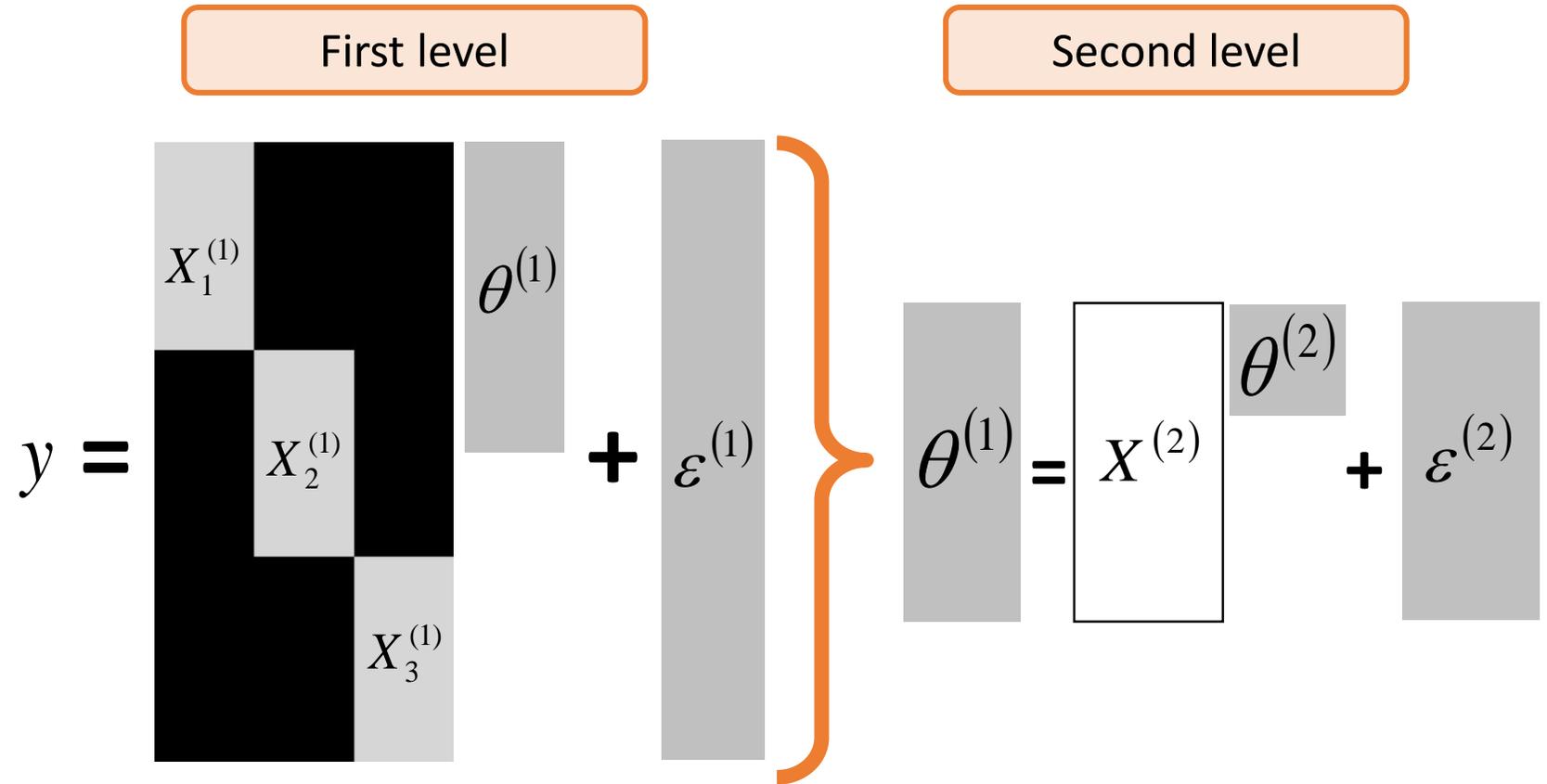
$$C_{\varepsilon}^{(i)} = \sum_k \lambda_k^{(i)} Q_k^{(i)}$$

What we don't know: distribution of parameters and variance parameters.

Hierarchical Model

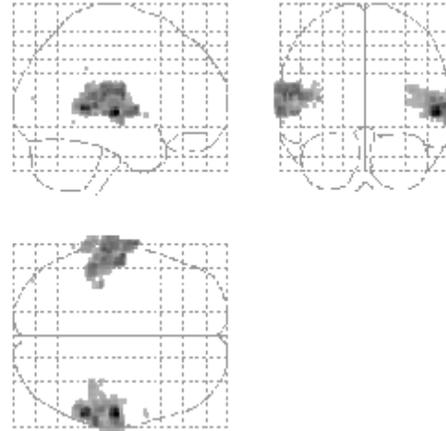
$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$

- (1) Within subject variance, $s_w(i)$
- (2) Between subject variance, s_b

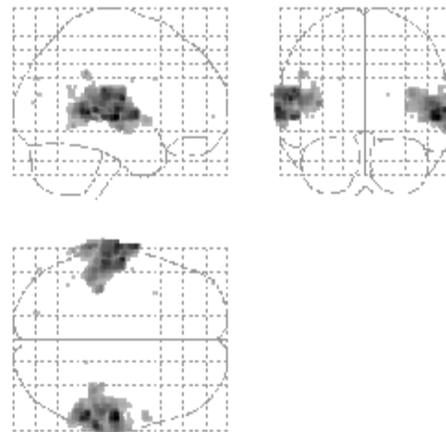


Example Results: Auditory Experiment

Summary
statistic



Hierarchical
model



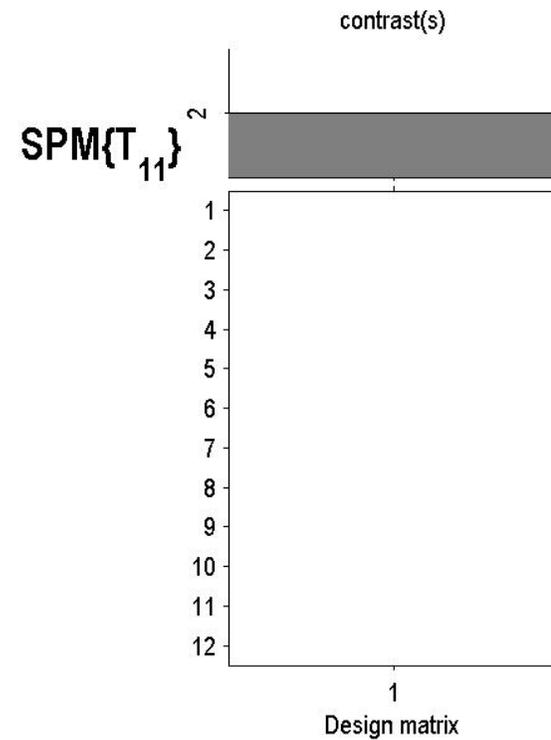
*Friston et al. (2004) Mixed effects
and fMRI studies, Neuroimage*

Summary Statistic vs. Hierarchical Model

- The summary stats approach is exact if, for each session/subject:
 - Within-subject variances are the same
 - First-level design (e.g. number of trials) are the same
- The summary stats approach is robust against typical violations (SPM book 2006; Mumford and Nichols, 2009, Neuroimage).
- We might use a hierarchical model in epilepsy research where number of seizures is not under experimental control and is highly variable over subjects.

Beyond the one sample t-test...

Second level



Multiple Conditions (within subjects)

Condition 1	Condition 2	Condition 3
Subject 1	Subject 1	Subject 1
Subject 2	Subject 2	Subject 2
...
Subject 12	Subject 12	Subject 12

Second level: One-way ANOVA within subjects

Multiple Conditions (between subjects)

Condition 1	Condition 2	Condition 3
Subject 1	Subject 13	Subject 25
Subject 2	Subject 14	Subject 26
...
Subject 12	Subject 24	Subject 36

e.g., effects of a drug

Second level: One way ANOVA between subjects
(or if only two conditions, a two-sample t-test)

Summary

- **Group inference usually proceeds with random effects analysis, not fixed effects analysis. Group effects are compared to between rather than within subject variability**
- **Hierarchical models provide a gold-standard for random effects group analysis, but are computationally intensive**
- **Summary statistics are a robust method for random effects group analysis when conditions are met**
- **If you want to contrast two conditions within subjects, you can use a one-sample t-test at the second level. If more conditions, you can use a one-way ANOVA. If different groups, you can use a between-subjects ANOVA or two-sample t-test**